RESEARCH ARTICLE

Application of Multi-Scale Modelling on Large Shear-Critical Reinforced Concrete Structural Systems Repaired with FRP Sheets

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Abstract: ***Background***: Numerical studies on the shear behavior of reinforced concrete (RC) structures repaired with fibre-reinforced polymer (FRP) sheets are relatively limited because of the complexity of the shear mechanisms. Almost all of the studies are conducted at the component-level due to the computational time and memory storage limitations associated with detailed finite element (FE) tools.

Methods: A multi-scale analysis framework was recently developed based on the substructuring technique for the nonlinear analysis of RC structures. In this study, the proposed framework was employed for modelling and analysis of structures strengthened with FRP. In this procedure, the repaired components were modelled in finer detail using a 2D FE program, while the rest of the structure was modelled with a computationally-fast frame analysis program. The sub-models were connected using a newly developed interface element, the F2M element, which satisfies equilibrium and compatibility conditions and provides an accurate shear stress distribution for cracked concrete at the interface. A practical and reliable method was presented to model FRP-related mechanisms for repaired components. Link elements were used to consider bond-slip effects and peeling-off phenomena of FRP sheets. The confinement enhancement of FRP was modelled by addition of an out-of-plane smeared component to the corresponding rectangular RC elements. Second-order material effects such as tension stiffening and crack spacing were taken into account using proper models.

Results and Conclusion: The application of the proposed modelling procedure was investigated by analyzing a RC frame with shear-critical beams repaired with FRP sheets. The analysis found that insufficient consideration of shear-related effects can lead to significant overestimations of strength and deformation capacity, and inaccurate predictions of structure behavior. Most frame analysis procedures, including plastic hinge and layered analysis approaches, require difficult assumptions and inputs to account for shear mechanisms which can significantly affect structural response. In general, the mixed-type analysis was able to accurately predict the behavior of the structure particularly in terms of stiffness, peak load, ductility, failure mode, and energy dissipation. The proposed method was capable of considering the effects of previous damage with the use of stress and strain history of the elements. In addition, the change in the damage mode prior and after the repair of the frame structure was captured accurately.

Keywords: Fibre reinforced polymer, multi-scale simulation, nonlinear finite element analysis, reinforced concrete, repaired structures.

INTRODUCTION

In recent years, several repair strategies have been developed for damaged RC structures. Among them, fibre-reinforced polymer (FRP) composites have proven to be an effective, convenient, and practical method of improving the performance of deficient structures, borne out in many experimental programs [1, 2] and real-world structures [3]. The advantages of FRP composites compared to other repaired methods such as infill walls, steel jacketing, and concrete caging are high strength-to-weight ratio, high corrosion resistance, improved fire resistance (if proper insulation is provided), versatile design, and easy installation.

Although a significant amount of research has been carried out on analyzing the behavior of RC structures repaired with FRP composites, most have been focusing on the flexural response of RC beams retrofitted with FRP sheets. Numerical studies on the shear strengthening of RC

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members using FRP sheets are relatively limited due to the complexity of the shear behavior. However, the brittle nature of shear failure which happens with little or no forewarning requires proper consideration, particularly when FRP sheets are used for shear strengthening. Al-Mahaidi *et al.* [4] performed two-dimensional FE analysis to investigate the behavior of shear deficient T-beams strengthened with web-bonded carbon FRP (CFRP) strips. The analysis assumed perfect bond between concrete and CFRP strips. The numerical response underestimated the peak load compared to the experimental results. Godat *et al.* [5] developed a three-dimensional FE model using ADINA [6] to simulate the behavior of FRP shear-strengthened RC beams. The model considered bond-slip effects between concrete and FRP using nonlinear link elements. The analysis showed good agreement with the experimental results. Sayed *et al.* [7] conducted a parametric study using a three-dimensional FE model in ANSYS [8] to identify the variables influencing the shear capacity of RC beams strengthened with FRP sheets. The interface between FRP and concrete sheets was modelled using contact elements. The computed peak load values agreed well with the experimental results. However, the load-deflection responses were not reported. The aforementioned shear models were developed for monotonic loading conditions. To investigate the response of structures under seismic loads, more comprehensive models with cyclic loading capabilities are needed. Li *et al*. [9] attempted to capture the cyclic behavior of shear walls strengthened with FRP plates using ABAQUS [10]. Spring elements were used to simulate the constraint on deformation provided by FRP sheets. Although the analysis was able to capture the failure mode and shape of the hysteresis response reasonably well, it significantly underestimated the peak load.

Furthermore, almost all the previous studies were performed at the component-level. There are few studies which attempted to model the entire structural system while considering the retrofitted components. Eslami and Ronagh [11] conducted a numerical study on an eight-storey RC building strengthened with glass FRP (GFRP) using SAP2000 [12]. The analysis considered the nonlinear behavior using flexural plastic hinges and neglected the shear-related effects. Galal and El-Sokkary [13] conducted a similar study in which each member was modelled as a linear elastic element with inelastic flexure-shear rotation springs located at the ends. With this approach, the force-displacement relationship for each member has to be input manually which requires expert users and significant amounts of time and effort. Garcia *et al*. [14] performed a two-dimensional FE analysis to evaluate seismic behavior of a two-storey RC building strengthened with CFRP. Beams and columns were modelled with nonlinear fibre elements. Although the model considered confinement enhancements due to FRP sheets, it neglected any possible debonding effects. In addition, to capture the damage accumulation, a calibrated stiffness degradation factor was introduced to the model which raises questions about the applicability of the method to other structural systems.

In this study, a unique modelling approach for multi-scaleanalysis of shear-critical RC structures strengthened with FRP under monotonic and cyclic loading conditions is proposed. The analysis is capable of considering the response of the structure, particularly in shear, at both the component-level and system-level. At the component level, damage effects and FRP-related mechanisms are considered in detail. The application of the proposed multi-scale method is demonstrated by analyzing a shear-critical frame repaired with FRP sheets and comparing the results against the experimental response.

ANALYSIS FRAMEWORK

The proposed multi-scale analysis procedure consists of three main components: an integration module, substructure modules, and interface modules. A brief description of each part along with references for further information are provided in the following sections.

Integration Module

An integration module, named Cyrus [15], was recently developed by the authors; it can combine different analysis tools while fully taking into account the interaction between the substructures. The retrofitted members along with other critical components of the structure can be modelled in a detailed FE program while the remainder of the structure can be modelled in another program using fibre beam elements. The coupled formulation of the analysis enables considering force redistribution due to stiffness changes between different components. Moreover, the integration module enables the use of the parallel processing technique to avoid the computational time and memory storage limitations associated with sequential single-platform analyses.

Substructure Modules

In this study, the critical components of the structure including retrofitted members were modelled and analyzed with VecTor2, a two-dimensional nonlinear FE software for RC structures. The program uses a smeared, rotating crack formulation based on the Modified Compression Field Theory (MCFT) [16] and the Disturbed Stress Field Model (DSFM) [17]. The MCFT and DSFM have been shown to be capable of accurately representing the behavior of RC particularly under shear [18, 19]. The program is able to consider material second-order effects such as compression softening due to transverse cracking, tension stiffening, localized stress disturbances, and shear slip along crack surfaces. In terms of analyzing the retrofitted components, detailed effects of FRP sheets such as strength and ductility improvement, concrete confinement enhancement, stresses at the interface, and peeling-off phenomena were considered in the model.

While the critical components are modelled in a detailed FE program, the remainder of the structure is modelled in a frame analysis program using lower-dimensional elements. Cyrus is compatible with two different analysis programs which provide computational and memory efficient frame-type elements: VecTor5 [20] and OpenSees [21]. OpenSees provides elastic beamelements which have rotational springs at the ends to represent the nonlinear behavior, suitable for flexure-critical structures. VecTor5 is a nonlinear frame analysis program based on the unbalanced force approach which can consider shear-related effects. In this study, to capture the shear behavior more accurately, VecTor5 was chosen for modelling the frame substructures.

Interface Modules

One of the main challenges in multi-scale simulation is the modelling of mixed-dimensional interfaces between the sub-models. Recently, a new interface element, named the F2M element, was developed by the authors for frame-membrane mixed-dimensional connection. The F2M element is a 2-node semi-deformable element that can fully transfer translational and rotational displacements at the interface. Compared to the traditional rigid interface method, the F2M interface element does not add any additional shear stiffness to the system and allows lateral expansion. Compared to other common types of interface methods such as multi-point-constraints (MPCs) approach and transition elements, the proposed method better takes into account the nonlinear behavior of the structure at the interface and provides a much more accurate shear stress distribution at the connection section.

A detailed description of the integration module and F2M interface element are provided in the Cyrus User's Manual [22]. Fig. (**1**) demonstrates parts of the proposed multi-scale modelling procedure for structural systems with repaired components.

MODELLING CONCRETE AND REINFORCEMENT

The analysis tool uses nonlinear elasticity based models rather than plasticity based models to compute the response in the material level for concrete and reinforcement components. The concrete compression pre-peak response was modelled using the Hognestad parabola. To account for the enhancement of concrete strength and ductility due to confinement, a modified Park-Kent model [23] was used for the compression post-peak response of concrete. The compression softening effect, representing the reduction of compressive strength and stiffness relative to the uniaxial compressive strength due to coexisting transverse cracking, was taken into account using the Vecchio and Collins model [24]. The hysteretic response of concrete was considered through use of the plastic offsets model proposed by Vecchio [25]. The resulting plastic offset strains, along with the area delineated by the hysteretic loops, are indicative of the internal damage and energy dissipation under cyclic loading. The model uses nonlinear Ramberg-Osgood formulations to compute the unloading response in the compression domain.

Two options are available for modelling reinforcing bars in the proposed analytical tool: the smeared option and the discrete option. Smeared reinforcement is modelled as a component of the concrete material within elements which can be four-noded rectangular or quadrilateral elements or three-node triangular elements depending on the geometry of the structure and required mesh. This option is suitable if the reinforcement is uniformly distributed over a large area (*e.g.*, shear reinforcement over a length of the beam). If the reinforcement is concentrated (*e.g.*, longitudinal reinforcement of a beam), then it is best to model the bars discretely using two-node truss elements. The discrete modelling of reinforcing bars enables the consideration of bond-slip effects between the reinforcement and concrete. Two-noded link elements can be used between truss elements and concrete elements to capture mechanisms related to bar slip. The hysteretic response of the reinforcement was represented using the Seckin model [26]. This model includes a linear elastic region followed by a yield plateau and strain hardening. The unloading and reloading response includes the Bauschinger effect. Potential buckling of the longitudinal bars was considered using the Dhakal and Maekawa model [27].

The analysis procedure employs a nonlinear elasticity approach, with the material strengths and post-peak responses dictated by the constitutive models discussed above; it does not rely on any plasticity-based failure hypotheses. The analysis continues until the secant moduli, displacements, or forces no longer reach convergence, after a certain number of iterations, as a result of structural capacity having been exceeded. The failure can be ascertained by

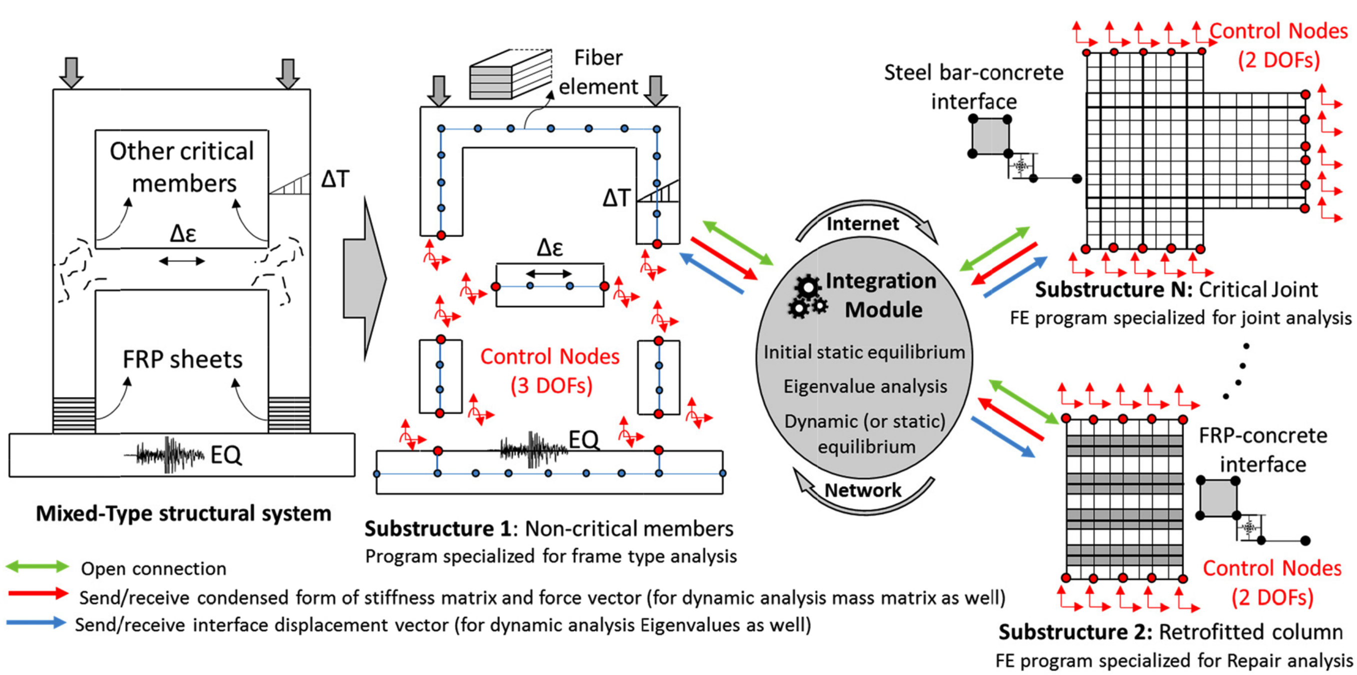

Fig. (1). Overview of the proposed multi-scale analysis approach.

examining several factors including computed load-deflection response, crack pattern, crack width, and stresses and strains in each component.

MODELLING FRP SHEETS

In the detailed FE sub-model, 2-node truss elements were used to model FRP sheets. Each node of the truss element has two translational degrees of freedom (DOFs). A uniform cross-sectional area computed from the thickness and tributary width of the FRP sheets is assigned to the truss elements. The stress-strain response is assumed linear-elastic up to the rupture of FRP in tension and zero stress in compression.

To accurately model bond-slip effects, link elements were used at the interface of RC rectangular elements and FRP truss elements. The link element is a 2-node non-dimensional element with a total of four translational DOFs. The element consists of two orthogonal springs connecting RC elements and FRP truss bars. One spring deforms tangentially to the FRP truss element, representing slip and stress of the bond. The other spring deforms radially to the truss element, representing radial displacements and stresses. The nodal displacements of the elements in the global X and Y coordinate system, [D], are transformed to deformations in the local directions of the FRP truss, [d], using a transformation matrix, [T]. The force in the tangential spring (f_t) is found by multiplying the bond slip (d_t) by the corresponding stiffness (k_t), and the bonded tributary area (A). The radial force (f_r) can be computed using a similar procedure. The tangential stiffness is determined from the bond-slip curve. The radial stiffness is given a very large value to prevent any displacement in the radial direction. Using the transformation matrix, the forces in the X and Y directions, [F], can be determined from the forces in the local directions. Based on the above-mentioned procedure, the equilibrium relationship of the link element in the X and Y directions is presented in Eq. (1) to Eq. (3).

$$[F] = A[K][D] \qquad (N) \tag{1}$$

$$[K] = [T]^T \begin{bmatrix} k_t & 0 \\ 0 & k_r \end{bmatrix} [T] \qquad (\frac{N}{m}) \tag{2}$$

$$[T] = \begin{bmatrix} -\cos\theta & -\sin\theta & \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta & -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

Several different bond constitutive relationships have been proposed to model the interface of concrete and FRP sheets. As shown in Fig. (**2**), the bond models can be categorized into three types: elastic models with cut-off [28], elastic-plastic models [29], and elastic models with softening [30-32]. In each model type, the ascending and descending branches can be linear or nonlinear depending on the model. Lu *et al.* [30] assessed the experimental data of 253 pull-out tests and demonstrated that the level of accuracy of linear models is similar to that of nonlinear models. Therefore, because of their simplicity, the linear models are more preferable for implementation in FE programs. Based on the literature, the elastic models with softening branchprovide the best correlation with the experimental results. Apart from the model shape, the bond behavior is defined with three parameters: maximum bond stress, slip corresponding to the maximum bond stress, and ultimate slip. Several factors can affect these parameters including: concrete compressive strength, FRP length and stiffness, width ratio of FRP and concrete, and adhesive material properties. In this study, the bond stress-strain relationship was computed using the Nakaba model [31]. This model was derived based on double-face shear type bond tests. The test variables included various types of fibre and concrete. The test results concluded that fibre stiffness and concrete compressive strength influence the maximum bond strength andthe shape of the stress distribution. However, the bond stress-strain behavior was not influenced by the type of fibre. Sato and Vecchio [33] conducted a series of bond tests and demonstrated that the Nakaba model can capture the bond-slip behavior reasonably well for fibre sheets with different lengths. This bilinear bond model, based on concrete fracture energy, is expressed by Eq. (4) to Eq. (7).

$$\tau_{bFy} = (54 f_c')^{0.19} (\frac{N}{m^2}) \tag{4}$$

$$S_{Fy} = 0.057 {G_F}^{0.5} \qquad (m) \tag{5}$$

$$G_F = (\frac{\tau_{bFy}}{6.6})^2 \qquad (\frac{N}{m}) \tag{6}$$

$$S_{Fu} = \frac{2G_F}{\tau_{bFy}} \qquad (m) \tag{7}$$

where τ_{bFy}, f_c', S_{Fy}, G_F, and S_{Fu} are the maximum bond shear stress, compressive strength of concrete, bond slip at the maximum shear stress, fracture energy of concrete, and ultimate bond slip, respectively. Because of the separation of FRP from concrete at the failure, Wong and Vecchio [34] recommended that the maximum FRP bond stress must be limited to the modulus of rupture of concrete, f_r.

$$\tau_{bFy} \leq f_r = 0.6 \times (f_c')^{0.5} (\frac{N}{m^2}) \tag{8}$$

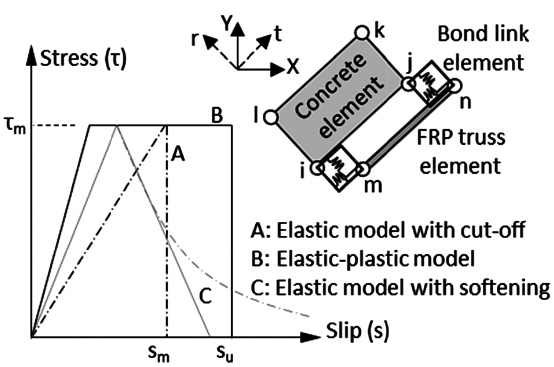

Fig. (2). FRP link element and different types of stress-strain relationship for bond.

Crack formation is an other factor which can influence the behavior of retrofitted RC members. FRP sheets can control crack width and reduce crack spacing. Also, premature debonding of the sheets, before attainment of the tensile strength of the FRP, can initiate at crack locations, and thus can begreatly influenced by crack spacing. In this study, the crack formation and tension stiffening effects were considered using the Sato and Vecchio model [33]. The model computes crack spacing and contribution of FRP to tensile strength by formulating the equilibrium at the crack location based on the Tension Chord concept [35]. The average tensile concrete stress contributed by FRP sheets is computed according to Eq. (9).

$$f_{c1} = \sum_{j=1}^{n} \rho_{F,j} E_{F,j} \Delta\varepsilon_F \cos^2 \theta_{F,j} \left(\frac{N}{m^2}\right) \quad (9)$$

where subscript "j" indicates a component of FRP sheets. E_F, $\Delta\varepsilon_F$, and θ_F are stiffness, difference between average strain and local crack strain, and angle between the FRP strip direction and principal tensile stress direction, respectively. ρ_F is the effective reinforcement ratio for the FRP sheet:

$$\rho_F = \frac{t_f}{R_{ef}} \quad (10)$$

where t_f is the thickness of the FRP sheet and R_{ef} is the distance from the FRP sheet over which the tension stiffening is effective.

$$R_{ef} = \frac{1}{f_t'} \sum_{j=1}^{n} \left(15.8 + 1.34\sqrt{t_{fj} E_{fj}}\right) \sqrt{G_{fj}} (m) \quad (11)$$

The strain difference, $\Delta\varepsilon_F$, is modelled by the curve formulated in Eq. (12).

$$\frac{\Delta\varepsilon_F}{\Delta\varepsilon_{F\,max}} = \frac{\varepsilon_{Fm}}{\varepsilon_{F1}} \times \frac{\alpha}{(\alpha - 1) + (\frac{\varepsilon_{Fm}}{\varepsilon_{F1}})^\alpha} \quad (12)$$

where ε_{Fm} is the average tensile strain in the FRP. $\Delta\varepsilon_{F\,max}$, ε_{F1}, and α parameters are calculated based on Eq. (13) to Eq. (16) which were derived by Vecchio and Sato.

$$\Delta\varepsilon_{F\,max} = \sqrt{G_F} \left\{ \frac{1340}{\sqrt{t_F E_F}} - 1.27 - \left[c_2 \left(\frac{S_r}{\cos\theta_F} - 640\right) \right]^4 \right\} \times 10^{-3} \quad (13)$$

$$\varepsilon_{F1} = \sqrt{G_F} \left[\left(\frac{185000}{t_F E_F} + 25 \right) \sqrt{\frac{\cos\theta_F}{S_r}} - 0.32 \right] \times 10^{-3} \quad (14)$$

$$\alpha = 2.7 - \left(\frac{S_r}{640 \cos\theta_F} \right)^2 \quad (15)$$

$$c_2 = \left[\frac{9.3}{(t_F E_F)^{0.05}} - 3.1 \right] \times 10^{-3} \quad (16)$$

where t_F and G_F are the thickness and fracture energy of the FRP. G_F is computed based on Eq. (6). S_r is the crack spacing parameter which is expressed by Eq. (17).

$$S_r = \frac{\lambda}{\frac{\sin\theta}{S_{rx}} + \frac{\cos\theta}{S_{ry}}} \quad (m) \quad (17)$$

where λ is the crack formation parameter which is 0.75. θ is the angle between horizontal axis and the principal tensile stress direction. S_{rx} and S_{ry} are the crack spacings perpendicular to the X and Y directions, respectively. Fig. (**3**) is a graphical demonstration of the contribution of FRP to tensile strength of concrete.

The confinement effects of FRP wraps are simulated with a smeared out-of-plane FRP component in the concrete element. The out-of-plane stresses and strains were utilized to compute the strength and ductility enhancements due to the confinement. The out-of-plane concrete strain is computed by Eq. (18).

$$\varepsilon_{cz} = \frac{-E_c}{E_c + \rho_{Fz} E_F} \left(\nu_{12} \frac{f_{c2}}{\overline{E}_{c2}} + \nu_{21} \frac{f_{c1}}{\overline{E}_{c1}} \right) \quad (18)$$

where E_c, $\overline{E}_c$, f_c, ν are the initial stiffness, secant stiffness, stress, and Poisson's ratio of concrete; E_F is the stiffness of FRP; and ρ_{Fz} is the FRP ratio in the Z direction (the out-of-plane direction of the FE model)which is equal to the volume of FRP sheets divided by the volume of concrete. Subscripts 1 and 2 indicate in-plane principal stress directions.

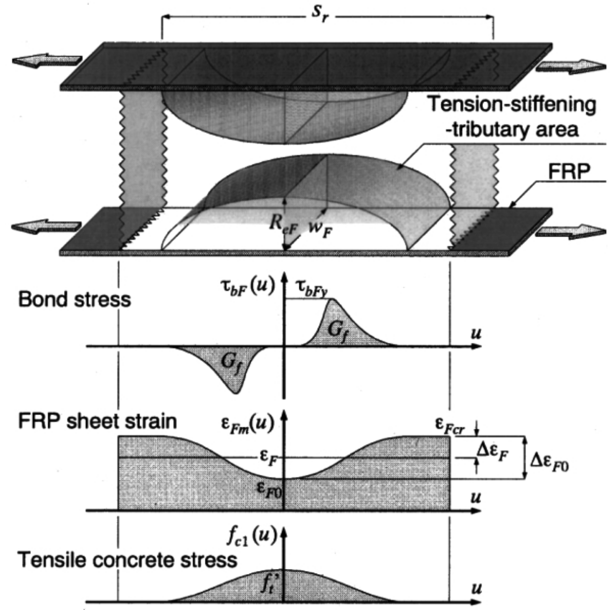

Fig. (3). Tension stiffening effects of FRP sheets [33].

Using the equilibrium the out-of-plane concrete compressive stress, f_{cz}, is determined from Eq. (19).

$$f_{cz} = -\rho_{Fz} \times f_{Fz} \left(\frac{N}{m^2}\right) \quad (19)$$

where f_{Fz} is the stress in the out-of-plane FRP sheet which is calculated based on Eq. (20) and must be less than the ultimate strength of FRP (f_{FU}).

$$f_{Fz} = E_F \varepsilon_{cz} \leq f_{FU} \left(\frac{N}{m^2}\right) \quad (20)$$

APPLICATION EXAMPLE: SHEAR-CRITICAL RC FRAME

Doung *et al.* [36] tested, a one-span two-storey RC frame with shear-critical beams under constant axial force and lateral displacement applied in a reversed-cyclic manner, as shown in Fig. (**4a**). The experimental program consisted of two test phases. In Phase A, the imposed lateral displacement was increased until significant diagonal shear cracks were observed in the beams. At this load stage, the shear crack width in the first-storey beam was 9 mm and the second-storey beam experienced a shear crack of 2 mm wide. Also, both the longitudinal and transverse reinforcement yielded in the beams, resulting in a flexural-shear damage mode. Then the frame was unloaded to zero displacement and reloaded in the reversed direction to the same displacement amplitude reached in the first half-cycle (44mm). In Phase B, the damaged beams were repaired by replacing unsound concrete, injecting epoxy into the cracks, and shear-strengthening the beams with FRP strips. After the repair process was completed, the frame was loaded in the reversed cyclic manner with the increasing displacements amplitude equal to the yield displacement (25mm) measured in the Phase A of the experiment.

Kim and Vecchio [37] performed a stand-alone FE analysis of the frame using VecTor2 software in 2008. Due to the detailed meshing requirements, the analysis was extremely time consuming and not applicable to larger structures. In addition, the analysis underestimated the energy dissipation and effective stiffness of the repaired structure. In this paper, to demonstrate the advantages of multi-scale simulation and deficiencies of frame-type analysis methods, two analysis studies were performed: a stand-alone frame analysis and a mixed-type analysis. The mixed-type analysis predicted the response of the frame structure with the same level of accuracy as the stand-alone detailed FE model in a significantly less time. In addition, the contribution of FRP to tensile strength of concrete was considered using the Tension Chord concept. While this resulted in a slightly higher peak strength in positive cycles compared to Kim and Vecchio's analysis, the pinching effects and effective stiffness of last cycles were predicted with a better accuracy. The following is a description of the model and computed results for each case study.

Stand-Alone Frame Analysis

In the stand-alone frame analysis, the entire structure was modelled using nonlinear layered beam elements. The joint panels were modelled with stiffened elements to avoid premature failure and account for the overlapping portions at the end zones. The computed load-deflection response is compared against the experimental results for Phase A in Fig. (**5**). In the first half cycle, the analysis overestimated the peak load and failed to accurately capture the behavior of the shear-critical beams due to highly nonlinear strain and stress distribution within the cracked regions. This resulted in a premature failure of the first storey beam whichcan be seen as a sudden strength drop in the load-deflection response. In the second half cycle, due to the failure of the first storey beam, the analysis significantly underestimated the stiffness and energy dissipation of the structure.

The stand-alone frame analysis was only limited to Phase A of the test, since the frame type procedures are unable to consider externally bonded FRP in detail and analyze repaired structures.

Mixed-Type FE Analysis

To capture the response of the shear-critical beams more accurately in Phase A and provide more detailed analysis of the repaired frame with FRP sheets in Phase B, a mixed-type analysis was conducted. As shown in Fig. (**4b**), the two shear-critical beams were modelled in a detailed FE program while the rest of the structure was modelled in a frame analysis software. The beams were modelled using rectangular concrete elements with a 40mm × 45mm mesh size. The transverse reinforcements were added as a smeared component to the concrete elements. The longitudinal reinforcements were modelled as discrete using truss elements. A total of 1190 elements were used to model the two critical beams. If a similar model for the entire frame structure were to be created, about3.5 times more elements would be required; this would result in much higher computational analysis time and memory usage without any improvement in accuracy. The multi-scale framework, Cyrus, was used to combine the sub-models and coordinate the mixed-type analysis. The sub-models were connected

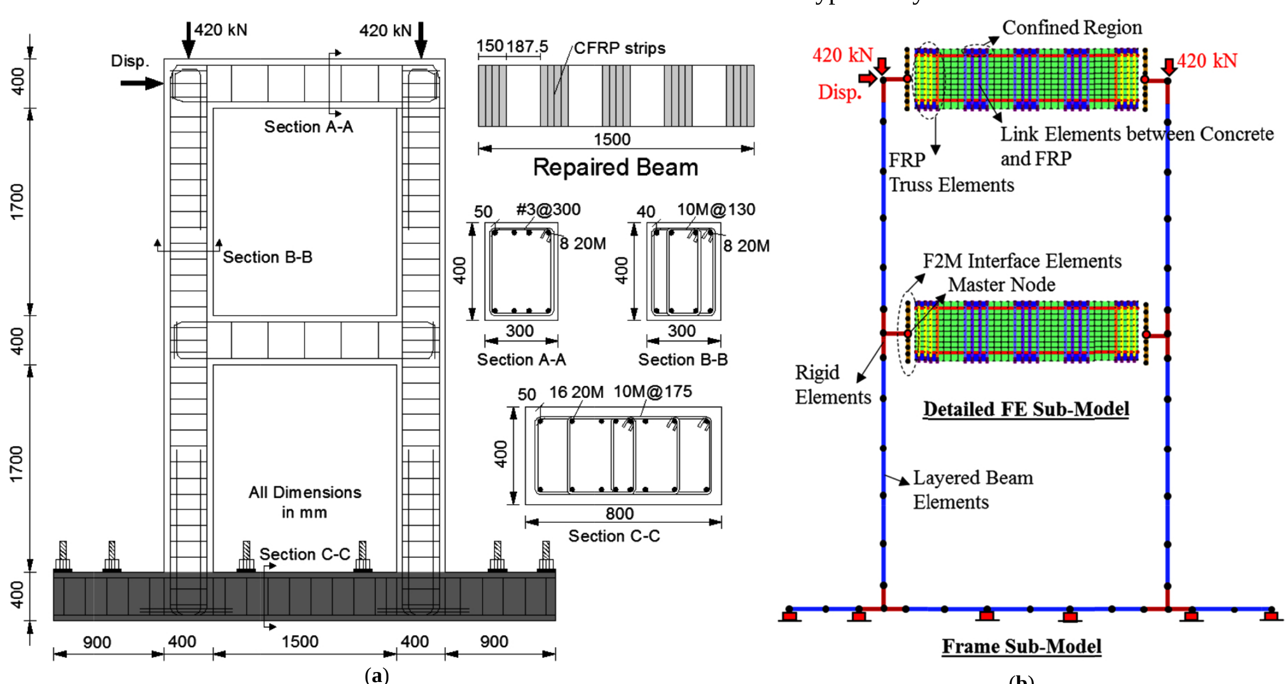

Fig. (4). (**a**) Details of the frame and repaired beam; (**b**) Mixed-type FE model of the frame.

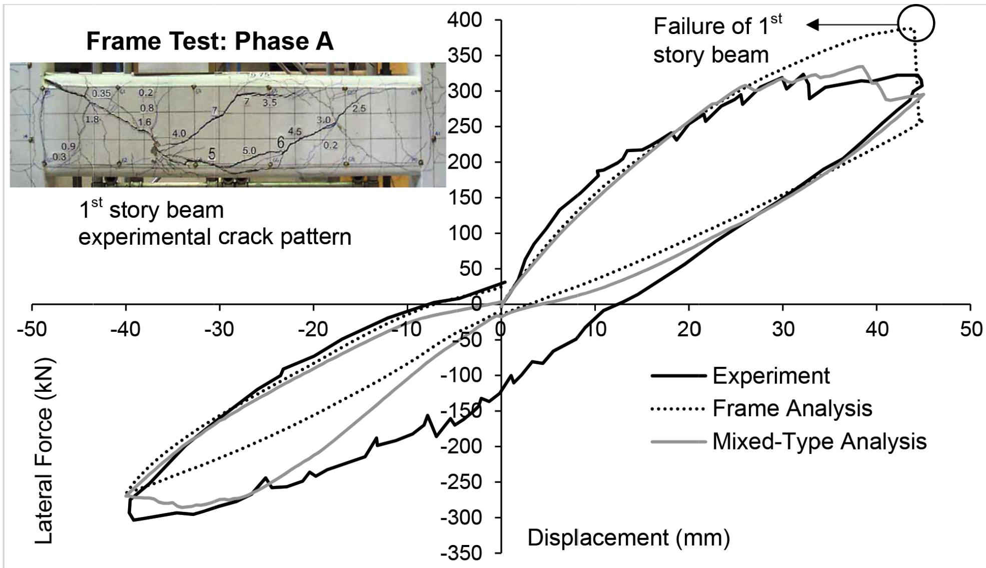

Fig. (5). Load-deflection response of the frame for Phase A.

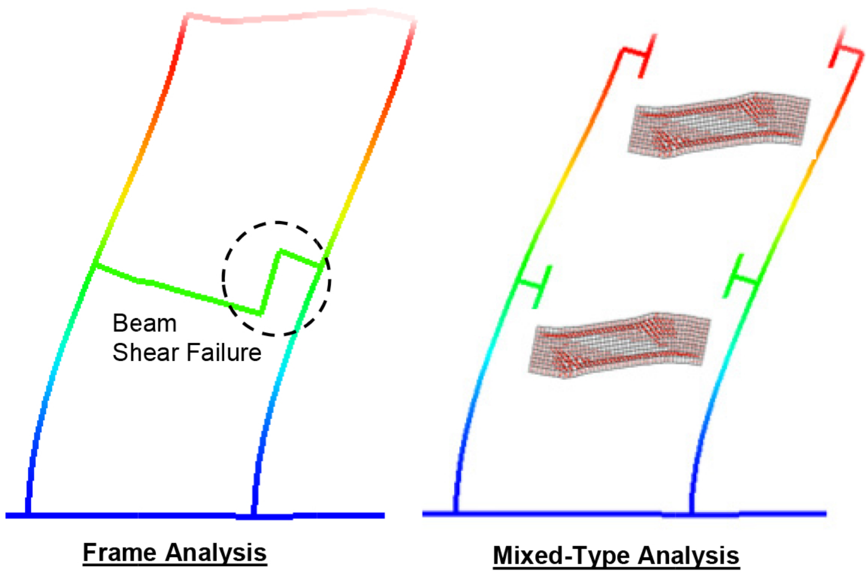

Fig. (6). Crack pattern and deflected shape of the frame.

with the F2M elements at the interface.

The mixed-type analysis load-deflection response is compared against the stand-alone frame analysis and experimental results in Fig. (**5**). The mixed-type analysis computed the stiffness, peak load, and energy dissipation with much better accuracy than the stand-alone frame analysis. In terms of damage mode, as shown in Fig. (**6**), the mixed-type analysis predicted large diagonal shear cracks at the ends of the beams which were continued to the mid-span at the top and bottom sections. A similar crack pattern was observed in the experiment. Both stand-alone analysis and mixed-type analysis underestimated the peak load in the reverse cycle and the pinching effect.

For analysis of the repaired frame in Phase B, the FRP sheets were modelled using truss elements which were connected with rectangular RC elements through link elements. For the concrete compressive strength of 43 MPa the maximum bond shear stress (τ_{bFy}) was computed as 4.36 MPa from Eq. (4). To account for the debonding of FRP from concrete at the failure, as recommended by Wong and Vecchio [34], τ_{bFy} was limited to the modulus of rupture of concrete which was 3.93 MPa from Eq. (8). Using Eq. (5) to Eq. (7) S_{Fy}, G_F, and S_{Fu} were calculated as 0.034 mm, 0.354 N/mm, and 0.180 mm, respectively. The confinement effects of FRP wraps were considered by addition of an out-of-plane component to rectangular elements located in the cover regions. To consider damage effects, Phase B of the analysis was started by reading and taking into account the stress and strain history of the elements from Phase A.

Fig. (**7**) compares load-deflection response of the analysis and experiment. The computed peak loads in the negative cycles, initial stiffness, and pinching effects correlated exceptionally well with the experiment. However, the analysis had a tendency to overestimate the peak loads associated with positive cycles. In both the experiment and the mixed-type analysis, a comparison between the response

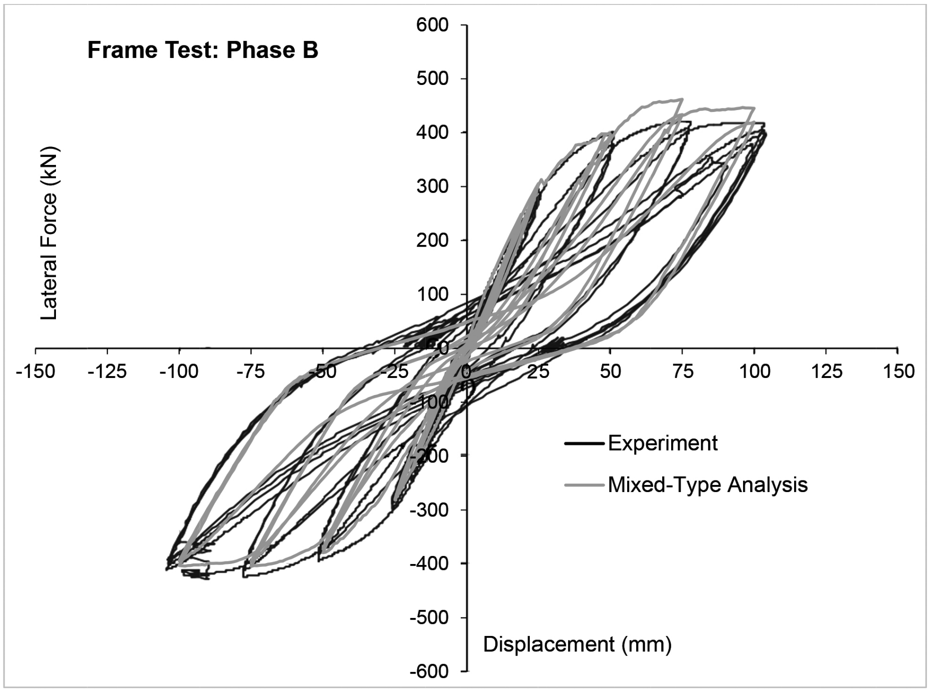

Fig. (7). Load-deflection response of experiment and mixed-type analysis for Phase B.

of Phase A and Phase B indicates that strengthening the shear-critical frame with FRP sheets greatly improved the ductility of the structure and changed the failure mode from shear to flexure with formation of hinges at the ends of the beams.

SUMMARY AND CONCLUSION

A multi-scale analysis framework was recently developed based on the substructuring technique for the nonlinear analysis of RC structures. In this study, the proposed framework was employed for modelling and analysis of structures strengthened with FRP. In this procedure, the repaired components were modelled in finer detail using a 2D FE program, while the rest of the structure was modelled with a computationally-fast frame analysis program. The sub-models were connected using a newly developed interface element, the F2M element, which satisfies equilibrium and compatibility conditions and provides an accurate shear stress distribution for cracked concrete at the interface.

A practical and reliable method was presented to model FRP-related mechanisms for repaired components. Link elements were used to consider bond-slip effects and peeling-off phenomena of FRP sheets. The confinement enhancement of FRP was modelled by addition of an out-of-plane smeared component to the corresponding rectangular RC elements. Second-order material effects such as tension stiffening and crack spacing were taken into account using proper models.

The application of the proposed modelling procedure was investigated by analyzing a RC frame with shear-critical beams repaired with FRP sheets. The analysis found that insufficient consideration of shear-related effects can lead to significant overestimations of strength and deformation capacity, and inaccurate predictions of structure behavior. Most frame analysis procedures, including plastic hinge and layered analysis approaches, require difficult assumptions and inputs to account for shear mechanisms which can significantly affect structural response. In general, the mixed-type analysis was able to accurately predict the behavior of the structure particularly in terms of stiffness, peak load, ductility, failure mode, and energy dissipation. The proposed method was capable of considering the effects of previous damage with the use of stress and strain history of the elements. In addition, the change in the damage mode prior and after the repair of the frame structure was captured accurately.

CURRENT & FUTURE DEVELOPMENTS

Currently, a research project is under development which intends to integrate experimental specimen with numerical models to perform pseudo-dynamic hybrid simulation using the proposed framework. Hybrid simulation testing of the repaired RC component will provide a more realistic response of the structure and will eliminate uncertainties in predicting the FRP-related mechanisms. As a result, a more accurate analysis of the interaction between the repaired component and other structural members of the system can be obtained.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

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REFERENCES

[1] H. Saadatmanesh and M. R. Ehsani, "RC beams strengthened with GFRP plates. I: Experimental study", *J. Struct. Eng.,* Vol. 117, pp. 3417-3433, 1991.

[2] J. Lombard, D.T. Lau, J.L. Humar, S. Foo and M. S. Cheung, "Seismic strengthening and repair of reinforced concrete shear walls, "In *Proc. 12th World Conf. on Earthquake Engineering*, New Zealand, 2000.

[3] S. A. Sheikh andS. M. Homam, "A Decay of performance of FRP-repaired concrete structures, " In *Proc. ISIS-SHM Workshop*, Winnipeg, Canada, 2004.

[4] R.Al-Mahaidi, K. Lee and G. Taplin, "Behavior and analysis of RC T-beams partially damaged in shear and repaired with CFRP laminates, "In *Proc. Structural Congress and Exposition, ASCE*, Reston, USA, 2001.

[5] A. Godat, K. Neale andP. Labossière, "Numerical modeling of FRP shear-strengthened reinforced concrete beams", *J. Compos. Constr.,* Vol. 11, pp. 640-649, 2007.

[6] K. J. Bathe, "Automatic dynamic incremental nonlinear analysis (ADINA) software", 1974.

[7] A.Sayed, X. Wang and Z.Wu, "Modeling of shear capacity of RC beams strengthened with FRP sheets based on FE simulation", *J. Compos. Constr.,* Vol. 17, pp. 687-701, 2013.

[8] J. A. Swanson, "ANSIS, engineering simulation software", Canonsburg, PA, USA, 1970.

[9] Z.J. Li, T. Balendra, K. H. Tan and K. H.Kong, "Finite element modeling of cyclic behavior of shear wall structure retrofitted using GFRP", *FRP7RCS Special Publication, ACI*, Vol. 230, pp. 1305-1324, 2005.

[10] "ABAQUS software user's manual", Dassault Systèmes Simulia Corp., Providence, RI, USA, 2003.

[11] A. Eslami and H.R. Ronagh, "Effect of FRP wrapping in seismic performance of RC buildings with and without special detailing – A case study", *Compos. Part B-Eng. J*, Vol. 45, pp. 1265-1274, 2013.

[12] "Analysis reference manual for SAP2000, ETABS, and SAFE", Computers and Structures, Inc., Berkeley, CA, USA, 2005.

[13] K. Galal and H.El-Sokkary, "Analytical evaluation of seismic performance of RC frames rehabilitated using FRP for increased ductility of members", *J. Perform. Constr. Fac.,* Vol. 22, pp. 276-288, 2008.

[14] R.Garcia, I.Hajirasouliha andK. Pilakoutas, "Seismic behaviour of deficient RC frames strengthened with CFRP composites", *Eng. Struct. J.,* Vol. 32, pp. 3075-3085, 2010.

[15] V. Sadeghian, F. J. Vecchio and O. Kwon, "An integrated framework for analysis of mixed-type reinforced concrete structures", *Comp. Dyn*, pp. 1-13, 2015.

[16] F. J. Vecchio and M. P. Collins, "The modified compression-field theory for reinforced concrete elements subjected to shear", *ACI Struct. J.,* Vol. 83, pp. 219-231, 1986.

[17] F. J. Vecchio, "Disturbed stress field model for reinforced concrete: Formulation", *J. Struct. Eng.*, Vol. 126, pp. 1070-1077, 2000.

[18] M. P. Collins, F. J. Vecchio and R. G. Selby, "The failure of an offshore platform", *Can. J. Civil Eng.*, Vol. 24, pp. 460-470, 1997.

[19] F. J. Vecchio, "Contribution of NLFEA to the evaluation of two structural concrete failures", *J. Perform. Constr. Fac.,* Vol. 16, pp. 110-115, 2002.

[20] S. Guner and F. J. Vecchio, "Analysis of shear-critical reinforced concrete plane frame elements under cyclic loading", *J. Struct. Eng.,* Vol. 137, pp. 834-843, 2011.

[21] F. T. McKenna, "Object-oriented finite element programming: frameworks for analysis, algorithms and parallel computing", PhD. Thesis, University of California, Berkeley, USA, 1997.

[22] V. Sadeghian and F. J. Vecchio, "Cyrus user's manual", University of Toronto, Toronto, Canada, 2014.

[23] B.D. Scott, R. Park and M.J.N. Priestley, "Stress-strain behavior of concrete confined by overlapping hoops at low and high strain rates", *ACI Struct. J.,* Vol. 79, pp. 13-27, 1982.

[24] F.J. Vecchio and M. P. Collins, "Compression response of cracked reinforced concrete", *J. Struct. Eng.,* Vol. 119, pp. 3590-3610, 1993.

[25] F. J. Vecchio, "Towards cyclic load modeling of reinforced concrete", *ACI Struct. J*., Vol. 96, pp. 193-202, 1999.

[26] M. Seckin, "Hysteretic behaviour of cast-in-place exterior beam-column-slab subassemblies", Ph.D. Thesis, Department of Civil Engineering, University of Toronto, Toronto, ON, Canada, 1981.

[27] R.P. Dhakal and K.Maekawa, "Modeling for post-yield buckling of reinforcement", *J. Struct. Eng.,* Vol. 128, pp. 1139-1147, 2002.

[28] U. Neubauer and F. S. Rostasy, "Bond failure of concrete fiber reinforced polymer plates at inclined cracks–experiments and fracture mechanics model, " In *Proc. of 4th Int. Symp. on FRP Reinforcement for RC Structures, ACI*, Michigan, United States, 1999, pp. 369-382.

[29] L.De Lorezis, B. Miller and A. Nanni, "Bond of fiber-reinforced polymer laminates to concrete", *ACI Mater. J.,* Vol. 98, pp. 256-264, 2001.

[30] X. Z. Lu, J. G.Teng, L. P.Ye and J. J. Jiang, "Bond–slip models for FRP sheets/plates bonded to concrete", *Eng. Struct. J.*, Vol. 27, pp. 920-937, 2005.

[31] K. Nakaba, T. Kanakubo, T. Furuta and H. Yoshizawa, "Bond behavior between fiber-reinforced polymer laminates and concrete", *ACI Struct. J*., Vol. 98, pp. 359-367, 2001.

[32] H. Ko, S. Matthys, A. Palmieri and Y. Sato, "Development of a simplified bond stress–slip model for bonded FRP–concrete interfaces", *Constr. Build. Mater. J.*, Vol. 68, pp. 142-157, 2014.

[33] Y. Sato and F. J. Vecchio, "Tension stiffening and crack formation in reinforced concrete members with fiber-reinforced polymer sheets", *J. Struct. Eng.,* Vol. 129, pp. 717-724, 2003.

[34] R. Wong and F. J. Vecchio, "Towards modeling of reinforced concrete members with externally bonded fiber-reinforced polymer composites", *ACI Struct. J.,* Vol. 100, pp. 47-55, 2003.

[35] W. Kaufmann and P. Marti, "Structural concrete: Cracked membrane model", *J. Struct. Eng.,* Vol. 124, pp. 1467-1475, 1998.

[36] K. V. Duong, S. A. Sheikh and F. J. Vecchio, "Seismic behavior of shear-critical reinforced concrete frame: Experimental investigation", *ACI Struct. J.,* Vol. 104, pp. 303-310, 2007.

[37] S. Kim and F. J. Vecchio, "Modeling of shear-critical reinforced concrete structures repaired with fiber-reinforced polymer composites", *J. Struct. Eng.,* Vol. 134, pp. 1288-1299, 2008.