NLFEARC
Look Both Ways Before Crossing

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Synopsis:

A critical look is taken at the state-of-the-art in nonlinear finite element analysis of reinforced concrete structures. In examining the results of recent prediction competitions, the accuracy of such analysis procedures is gauged. Reasons for caution when applying nonlinear analysis methods are then identified and discussed. Finally, the results of a test program involving shear critical beams are presented in support of the contention that the behaviour of reinforced concrete is still not well understood. The tests represent a good challenge for validating current procedures.

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INTRODUCTION

Nonlinear finite element analysis of reinforced concrete (NLFEARC) has taken tremendous strides forward since initial applications about 40 years ago. Much research activity has occurred in the realm of constitutive modelling of reinforced concrete behaviour and in the development of sophisticated analysis algorithms. Occurring at the same time, and no less significant, were quantum leaps in computing technology, greatly expanding the size of analyses that can be considered and greatly reducing the time required for solution.

The state-of-the-art has progressed to the point where NLFEA is close to being a practical, every-day tool in the arsenal of design office engineers. No longer the purview of the researcher, it is finding use in various applications; many relating to our aging infrastructure. NLFEA procedures can be used to provide reliable assessments of the strength and integrity of damaged or deteriorated structures, or structures built to previous codes, standards or practices deemed to be deficient today. They can be valuable tools in assessing the expected behaviour from retrofitted structures, or in investigating and rationally selecting amongst various repair alternatives. And, in situations that haven't turned out well, NLFEA procedures are finding applications to forensic analyses and litigations that follow. Not far down the road, they will likely form the main engine in computer-based automated design software, in a form likely invisible to the user.

THE QUESTION OF ACCURACY

Despite the increasing sophistication of NLFEA tools, users must be ever mindful of the question of how accurately and reliably do they represent the behaviour of reinforced concrete structures. In this regard, it is useful to examine the results of three recent 'prediction competitions'.

At the 1981 IABSE Symposium in Delft, a 'blind' competition was organized involving four panels tested in a comprehensive research program then underway at the University of

Toronto. The panels were orthogonally reinforced, and subjected to uniform stress conditions: seemingly a very simple problem to model and analyse. The results of these panel tests were not disclosed prior to analysts submitting their *predictions* of strength and load-deformation response. Approximately thirty entries were received, many from the leading researchers in the field at the time. Shown in Figure 1 is the range of responses for Panel C, one of the better predicted of the four panels. The analysis results submitted showed a wide variation in predictions of the panel's shear strength, and an even wider divergence in computed load-deformation responses (1). Clearly, the collective ability to model nonlinear behaviour of reinforced concrete, particularly in shear-critical conditions, was not well advanced.

More recently, in 1995, the Nuclear Power Engineering Corporation of Japan (NUPEC) staged a prediction competition involving a large-scale 3-D shear wall subjected to dynamic cyclic loading (2). The flanged shear wall exhibited highly nonlinear behaviour before sustaining a sliding shear failure along the base of the web. One facet of the competition required estimates of the ultimate strength, and corresponding displacement, of the wall as determined from static push-over analyses. Again, over thirty sets of predictions were received; the results are summarized in Figure 2. The predictions of strength, as a group, showed better correlation than were seen with the Toronto panels; however, the deformation estimates still showed large scatter. Nevertheless, it could be concluded that the ability of NLFEA to accurately capture the behaviour of reinforced concrete had measurably advanced. It should be noted, however, that this was not a completely 'blind' competition since some of the test results had been disclosed prior to the competition.

Finally, we have a series of large-scale columns tested at the University of California at San Diego. These columns were the subject of an informal competition organized recently by ASCE-ACI Committee 447. Many of the analyses undertaken pursuantly are documented in papers contained within this set of proceedings. As one reads through the papers, it can be noted that: i) a number of quite different analysis approaches were taken; ii) predictions of strength and pre-peak response generally correlate well with the experimental results; and iii) predictions of post-peak response are generally not as accurate and still require further attention. [Bear in mind, once again, that this was not a blind competition. Analysts have the opportunity to *calibrate* parameters, *optimize* material models, and *refine* analyses.]

Nevertheless, it is an inescapable conclusion that our ability to accurately model the behaviour of reinforced concrete structures has seen tremendous improvement over the past 20 years. It is approaching a stage of development where we may be inclined to proceed with a certain degree of confidence.

REASONS FOR CAUTION

Despite seemingly significant advancements in our ability to accurately model the response of reinforced concrete, the users of NLFEA procedures need to be mindful of several issues and caveats:

Diversity of Theoretical Approaches

A number of rather diverse approaches exist for NLFEA modelling of reinforced concrete structures. Among those available are models built on nonlinear elasticity, plasticity, fracture mechanics, damage continuum mechanics, endochronic theory or other hybrid formulations. Cracking can be modelled discretely, or using smeared crack approaches; the latter can range from fully rotating crack models, to fixed crack models, to multi-orthogonal crack models, to hybrid crack models. Some approaches place heavy emphasis on classical mechanics formulations; others draw more heavily on empirical data and phenomenological models. It can generally be said of any approach that it will be more suited to certain structure/loading situations and less so to others. No one approach performs well over the entire range of structural details and loading conditions encountered in practice.

Diversity of Behaviour Models

Reinforced concrete structures, particularly in their cracked states, are dominated in their behaviour by a number of second-order mechanisms and influencing factors. Depending on the particular details and conditions prevailing, a structure's strength, deflection, ductility and failure mode may be significantly affected by mechanisms such as: compression softening due to transverse cracking, tension stiffening, tension softening, aggregate interlock and crack shear slip, rebar bond slip, rebar dowel action, scale effects, and creep and shrinkage, to name a few. For each of these, a number of diverse formulations can exist. In the case of tension stiffening, for example, the stiffening effect can be ascribed to a post-cracking average tensile stress in the concrete or, quite differently, to the constitutive response of the rebar steel. The user of NLFEA software must be aware of what mechanisms are likely to be significant in the problem at hand, be certain that it is included in the analysis model, and have some confidence that the model being used is reasonably accurate.

Incompatibility of Models and Approaches

The formulation and calibration of a concrete behaviour model, as it is being developed, is often dependent on the particular analysis methodology being used. As a consequence, some models cannot be randomly transplanted from one analysis approach to another, or haphazardly combined with other models. Often, they are developed in tandem with other complementary material models, or analysis approaches, and should not be separated. As a case in point, consider the observed and predicted behaviour of Panel PV19 (3); this was, in fact, Panel C from the 1981 Delft Competition, represented in Figure 1. An important mechanism in influencing the shear strength and deformation response of this element was the softening of the concrete in compression due to transverse cracking, with the panel eventually sustaining a concrete shear failure. Shown in Figure 3 are the predicted responses obtained using the compression softening models of Vecchio and Collins implemented in a rotating crack formulation, and of Maekawa implemented in a fixed crack formulation (i.e., each correctly matched with crack model for which it was first developed). Both provide equally good simulations of response. The Vecchio-Collins formulation slightly over-estimates strength and slightly under-estimates ductility. Conversely, the Maekawa formulation slightly under-estimates strength and slightly over-estimates ductility. Either one, however, is certainly well within the margins of accuracy we can hope to achieve with NLFEA. But consider what happens if one implements the Vecchio-Collins model into a fixed crack formulation, or if one uses the Maekawa model in a rotating crack formulation. In both cases, the results are much less satisfactory; strength, ductility and failure mode are subject to significant miscalculation. As it turns out, a hybrid formulation between fixed and rotating crack models (4) provides the most accurate simulations in this case.

Experience Required

Use of NLFEA for modelling and analysis of reinforced concrete structures requires a certain amount of experience and expertise. Unlike, say, the use of plane section analysis techniques to calculate the flexural strength of a beam cross section, the application is rarely straightforward. Decisions made with respect to modelling of the structure and selection of material behaviour models will have significant impact on the results obtained. Again, unlike sectional analysis techniques, two analysts may very well get widely diverging results when modelling the same structure using the same analytical model and the same software. This is well known to instructors of graduate courses on NLFEARC, even when considering as simple a case as a simply supported beam with a mid-span load. Decisions made regarding mesh layout, type of element used, representation of reinforcement details, support conditions, method of loading, convergence criteria, and selection material behaviour models, will result in significant scatter in the results. Add to this the increased likelihood of errors in input due to the relative complexity of NLFEA. [The scatter of results is only marginally reduced when the group of analysts is reduced to those actually working in the field of nonlinear analysis and modelling of RC structures!] Hence, there is ample explanation for the significant difference in accuracy of results obtained in 'blind' competitions as opposed to those obtained when the desired results are known in advance.

Reams of Results

NLFEA invariably produce huge quantities of data, typically spawning output files of several megabytes or larger in size for each load stage. Information may be provided on: stresses and strains at each integration point of each element, both with respect to local and principal axes, both for the element and for the concrete component; nodal displacements; sectional forces per unit width at each integration point; reactions; reinforcement stresses and strains; stiffness matrix coefficients, etc. Considering that typical problems can involve tens of thousands of degrees of freedom, the total amount of data quickly escalates to the point where the use of post-processors is virtually essential. Even then, the analyst must have an awareness of what to look for and how to interpret it. Still, even with the use of sophisticated post-processors and graphics capabilities, there is the possibility for misinterpretation. After all, it was the incorrect processing and interpretation of FEA data that was directly responsible for the collapse and loss of the $500M Sleipner A offshore platform (5).

Incomplete Knowledge

To paraphrase Bob Dylan from one of his songs, 'we should know our song well before we sing it'. So, too, we should accept that we still do not understand well, let alone have accurate models for, many aspects of reinforced concrete behaviour. [See 'Challenging Set' below.] Applications of NLFEA should be done with a healthy dose of caution and scepticism. Wherever possible, analysis software and models should be validated or calibrated against benchmark tests involving specimens of similar construction and loading details, dependent on mechanisms anticipated to be significant in the analysis problem at hand (as far as this can be done). And wherever possible, results should be supported by analyses based on different models or approaches.

Research Philosophy

Lastly, it must be said that the research community, and associated technical committees, have failed the profession in some respects. Many working in the area have directed their efforts to developing sophisticated models and methods of analysis, in many cases basing their work on esoteric models or rigorous application of classical mechanics approaches not directly suited to reinforced concrete. Advancements have been made in developing the NLFEA concepts and methodologies, but often with reinforced concrete being merely the *application*. Unfortunately, reinforced concrete is complex and stubborn material that sometimes refuses to act according to accepted rules of mechanics or even common logic. Instead, efforts should be refocused toward better understanding and modelling reinforced concrete behaviour, with finite element analyses being merely the *tool*. Certainly, however, there is room and need to advance on both fronts.

A CHALLENGING SET OF TEST RESULTS

To reinforce the notion that we still do not know enough about the behaviour of reinforced concrete, and to provide a private challenge to those who might think otherwise, consider two series of beams tested by Angelakos et.al. (6).

The first series involved five beams each of which was 6000 mm in length, 1000 mm in depth, 300 mm wide, reinforced with approximately 1.0 percent longitudinal reinforcement, containing no transverse reinforcement, and subjected to a monotonically increasing load applied at the midspan. [Beams DB120, DB130, DB140, DB165 and DB180; see Figure 4 for beam details.] The only variable was the compressive strength of the concrete, ranging from 20 MPa to 80 MPa. These beams failed in a shear critical manner, upon the formation of the first web shear crack. Current design code formulations would say that the shear strength of these beams is directly proportional to a 'concrete contribution' related to the tensile strength of the concrete, which in turn is normally related back to the compressive strength (typically, the tensile strength is taken as proportional to the square of the compressive strength). Hence, the 80 MPa beam could be expected to have a shear strength of close to double that of the 20 MPa beam. Finite element analyses could also be expected to produce similar trends in predicted strength, since concrete tensile strength is an over-riding parameter in most analyses in such cases. Shown in Figure 5 are the load-deformation responses measured experimentally. Note that there is little difference in the strengths and pre-peak deflection responses obtained; certainly nothing approaching the doubling of strength anticipated. In fact, the 80 MPa beam exhibits a shear strength lower than the 20 MPa beam. At work are mechanism relating to smoothness of the fracture plane, aggregate interlock mechanisms, and crack slip mechanisms.

A second series of beams from the same test program involved four beams similar in dimensions and loading to the first [Beams DB120M, DB140M, DB164M and DB 180M; see Figure 4]. The principal difference was that these beams contained the near-minimum amount of shear reinforcement, at an amount of 0.22%. Again, the compressive strength of the concrete ranged from 20 MPa to 80 MPa. Shown in Figure 6 are the load-deformation responses recorded. Note that here, a small amount of shear reinforcement had a substantial influence on the strength and failure mechanisms observed. Although the higher strength concrete beams did exhibit a higher shear strength, there was still a good deal of perplexing behaviour observed. See Angelakos et al (2001) for a more complete description of the test program, and a more thorough discussion of results and significance.

These two series of test results will provide a stringent test of any NLFEA model. Analysts are encouraged to formulate all models, and select all design models and parameters in advance of any 'preliminary' analysis, and to conduct the analyses in a group and only once! *Know your song well before you sing it!*

CONCLUSIONS

The state-of-the-art in the nonlinear finite element analysis of reinforced concrete structures has advanced significantly in recent years. The speed and accuracy of analyses have progressed to the point where NLFEA software are close to being an indispensable design office tool, useful in many types of practical applications. In coming years, they may well form the heart of advanced software for automated design, albeit in a form transparent to the user.

However, NLFEA models and procedures for reinforced concrete remain complex, fragmented, and fraught with dangers. The user of these procedures must be experienced, cautious and somewhat sceptical.

Future work must be directed at developing improved material behaviour models that more accurately represent the behaviour of reinforced concrete under the diverse conditions encountered in practice. As well, efforts are needed in reconciling the incompatibilities of models and approaches, and in reducing the potential for errors in modelling, analysis and interpretation of results.

REFERENCES

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NOTATION

f'_c = compressive strength of concrete cylinder at 28 days
f_y = yield strength of reinforcement
ε_o = concrete strain at peak compressive stress
γ = shear strain
ρ = reinforcement ratio
τ = shear stress

KEY WORDS

Analysis
Finite element
Nonlinear
Shear
Panel tests
Shear beams
Concrete structures

FIGURE CAPTIONS

Figure 1: Delft Competition Results

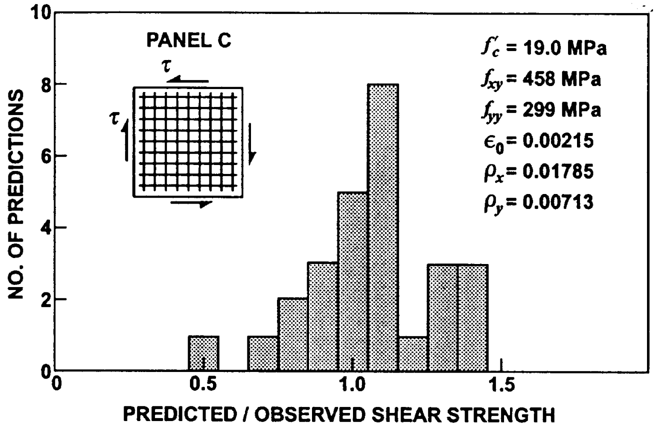
Figure 2: NUPEC Wall Prediction Results

Figure 3: Predictions of Panel PV19

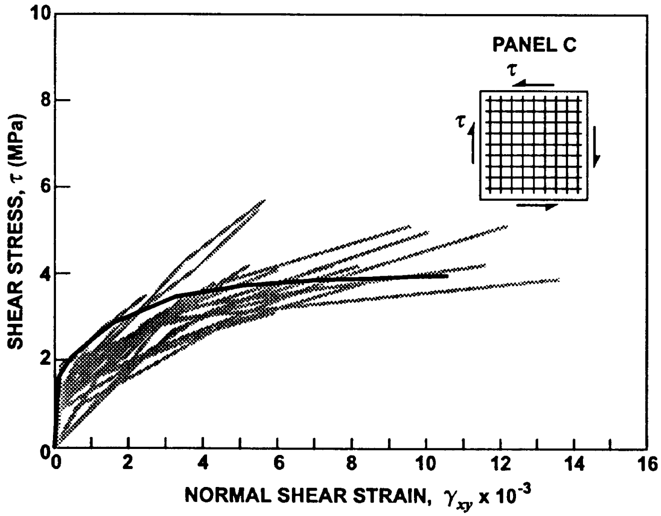
Figure 4: Details of Angelakos Beams

Figure 5: Results of Series I Beams

Figure 6: Results of Series II Beams

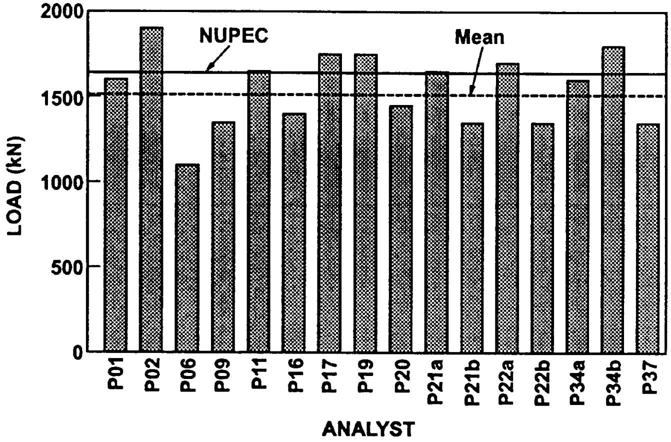

PANEL C
τ
τ
f'c = 19.0 MPa
fxy = 458 MPa
fyy = 299 MPa
ε0 = 0.00215
ρx = 0.01785
ρy = 0.00713
NO. OF PREDICTIONS
PREDICTED / OBSERVED SHEAR STRENGTH
0
2
4
6
8
10
0
0.5
1.0
1.5

(a)

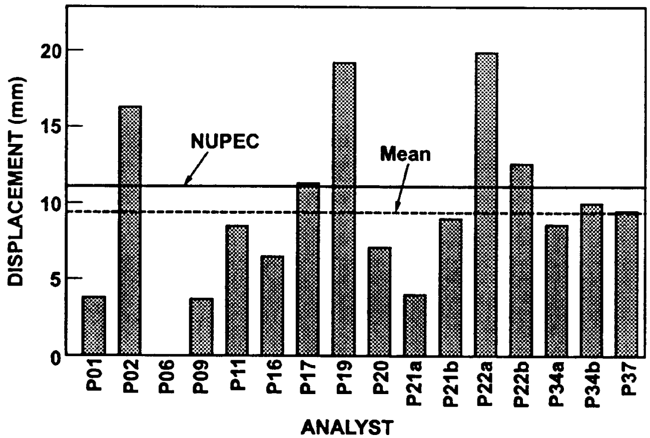

PANEL C
τ
τ
SHEAR STRESS, τ (MPa)
NORMAL SHEAR STRAIN, γxy x 10-3
0
2
4
6
8
10
0
2
4
6
8
10
12
14
16

(b)

Fig. 1

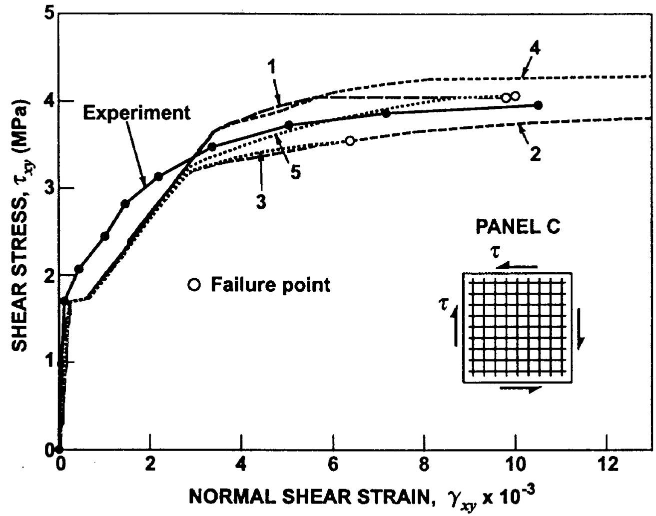

2000
1500
1000
500
0
LOAD (kN)
NUPEC
Mean
P01
P02
P06
P09
P11
P16
P17
P19
P20
P21a
P21b
P22a
P22b
P34a
P34b
P37
ANALYST

(a)


20
15
10
5
0
DISPLACEMENT (mm)
NUPEC
Mean
P01
P02
P06
P09
P11
P16
P17
P19
P20
P21a
P21b
P22a
P22b
P34a
P34b
P37
ANALYST

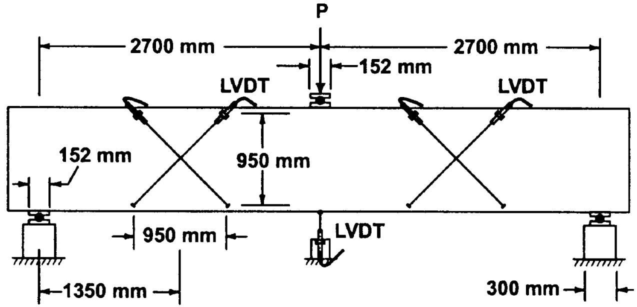
(b)

Fig. 2

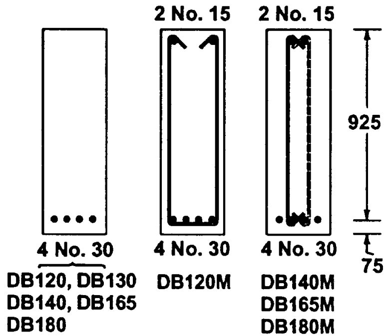


Analysis No.	Constitutive Model	Crack Model	τ_u (MPa)	γ_u (x 10^{-3})	Failure Mode
1	Vecchio-Collins	Rotating	4.05	9.80	Conc. Shear
2	Maekawa	Fixed	3.85	16.51	Conc. Shear
3	Vecchio-Collins	Fixed	3.55	6.42	Conc. Shear
4	Maekawa	Rotating	4.45	73.79	Rebar Yield
5	DSFM	Hybrid	4.07	10.02	Conc. Shear
Experiment			3.96	10.51	Conc. Shear

fig. 3



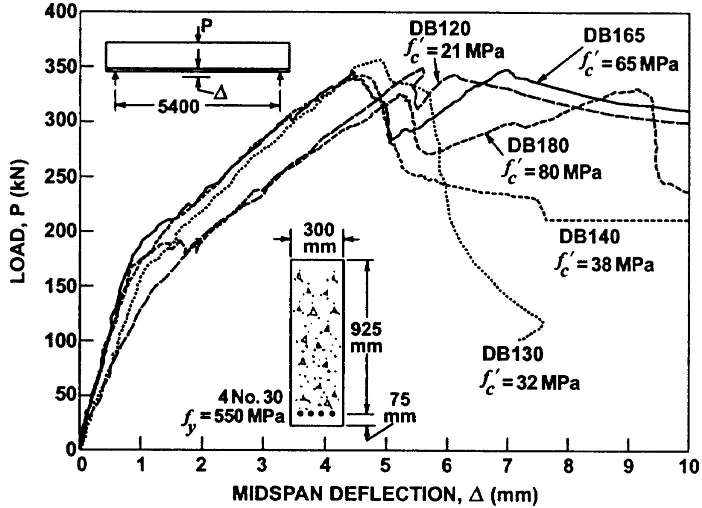
Span dimensions and LVDT locations



Bar	No. 30	No. 15	#3
Area (mm^2)	700	200	71
f_y (MPa)	550	483	508

#3 Stirrups either:
- One leg at 300 mm, or
- Two legs at 600 mm

Fig. 4



LOAD, P (kN)
400
350
300
250
200
150
100
50
0
P
Δ
5400
DB120
$f_c' = 21$ MPa
DB165
$f_c' = 65$ MPa
DB180
$f_c' = 80$ MPa
DB140
$f_c' = 38$ MPa
DB130
$f_c' = 32$ MPa
300 mm
925 mm
75 mm
4 No. 30
$f_y = 550$ MPa
0
1
2
3
4
5
6
7
8
9
10
MIDSPAN DEFLECTION, Δ (mm)

Fig. 5

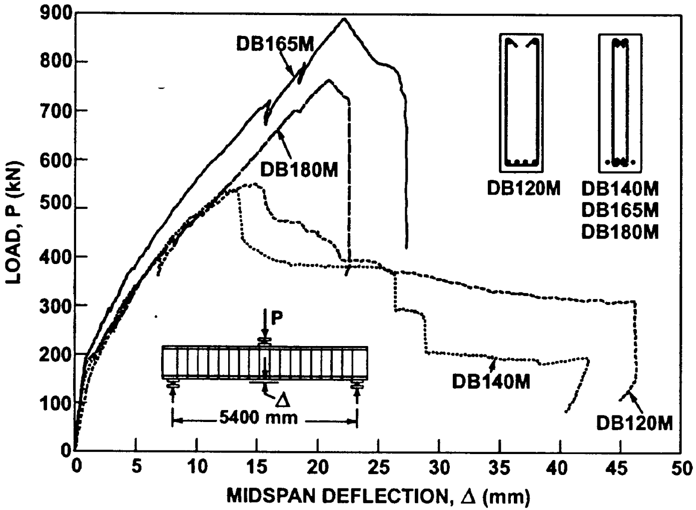

900
800
700
600
500
400
300
200
100
0
0
5
10
15
20
25
30
35
40
45
50
LOAD, P (kN)
MIDSPAN DEFLECTION, Δ (mm)
DB165M
DB180M
DB140M
DB120M
DB120M
DB140M
DB165M
DB180M
P
Δ
5400 mm

Fig. 6